

# Zitterbewegung-like effect near the Dirac point in metamaterials and photonic crystals

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We present a physical explanation of Zitterbewegung-like effect near the zero-refractive-index point in a metamaterial slab in this paper. Between the negative and positive refractive index regions centered at the zero-refractive-index point, the transmittance spectrum distribution of the metamaterial slab is asymmetrical. When a symmetrical pulse propagates through the metamaterial slab, its transmitted spectrum becomes asymmetrical due to the asymmetry of the transmittance spectrum of the slab, leading to a transmitted pulse with an asymmetrical temporal shape. The asymmetry manifests a kind of temporally tailed oscillations, *i.e.*, the Zitterbewegung-like effect. Further, the effect of the temporal and spatial widths of pulse, and the thickness of metamaterial slab on the tailed oscillations of the transmitted pulse has also been discussed. Our results agree well with what the other researchers obtained on the strength of relativistic quantum concepts; however, the viewpoint of our analysis is classical and irrelevant to relativistic quantum mechanics.

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## I. INTRODUCTION

The propagation of electromagnetic waves near the Dirac point (DP) in photonic crystals and metamaterials has been investigated intensively in recent years [1–11]. With some special structure designs of both kinds of artificial materials, two energy bands touch each other, forming a pair of cones, namely, so-called “Dirac point” in optics. For a monochromatic electromagnetic wave propagating near the DPs, some abnormal transmission properties have been discovered, such as pseudo-diffusive scaling [3–6] and conical diffraction [7]. The propagation of optical pulses with the central frequency at the DP in a photonic crystal or metamaterial slab has also been investigated by some research groups who show that temporally tailed oscillations of the transmitted pulses should appear [8, 11]. In their papers, this kind of oscillation is called Zitterbewegung (ZB) effect for photon.

The existence of ZB effect was first proposed by Erwin Schrödinger for relativistic electrons in free space, and the interference between the positive and negative energy states was considered as the origin [12]. ZB effects in crystals, superconductors, semiconductor nanostructures, graphene, and other physical systems have also been demonstrated [13–27]. And ZB effect for photon has been proposed in two-dimensional photonic crystal for the first time by simulation analysis by Zhang [8], in which he used a plane wave with Gaussian shape in time domain, passing through an open slit, to excite the photonic crystal slab. In another paper for the case of sonic crystals, Zhang and his co-worker demonstrated the ZB effect in experiment and mentioned that the origin of this effect was the interference between two linear modes around the DP [9]. Shortly thereafter, DP in the negative-zero-positive index metamaterial (NZPIM) was first proposed by Wang *et al.* [10]. Thereupon the ZB effect of optical pulses in NZPIM was characterized [11]. Both the research groups

named the temporally tailed oscillations of the transmitted pulses as ZB effect and thus explained it on the strength of some concepts of the relativistic quantum mechanics, such as DP and Dirac equation. This is undoubtedly an interesting and helpful understanding of the underlying mechanism of the effects in photonic crystal and metamaterial. However, we think that it may be not suitable to use ZB directly as the name of the tailed oscillations of optical pulses. So, here, we introduce Zitterbewegung-like (ZB-like) instead.

In this work, based on another point of view which is irrelevant to the relativistic quantum mechanics, we present a physical explanation of temporally tailed oscillations (or ZB-like effect) of the pulses propagating near the zero-refractive-index point (called DP in Refs. [10, 11]) in NZPIM using the spatio-temporal filtering analysis. Due to the asymmetrical transmittance spectrum distribution between both sides of the zero-refractive-index point, a symmetrical pulse passed through a NZPIM slab undergoes an asymmetrical filtering effect, which leads to the asymmetrical temporal shape of the transmitted pulse, in other words, the tailed oscillations. Further, the effect of the temporal and spatial widths of pulse, and the thickness of the NZPIM slab on the tailed oscillations of the transmitted pulse can be characterized well. The analysis method we use is similar to that of previous works [8–11] on dealing with some detail problems, however, our viewpoint, based on classical spectral filtering analysis that is not related to the relativistic quantum mechanics, is totally different from theirs. Even more interesting is the fact that the results we obtained are in good agreement with those in Refs. [8] and [11]. Using this analysis method, ZB-like effect in two-dimensional photonic crystals could also be explained.

## II. MODEL AND THEORY

We take the homogenous NZPIM slab as the analysis sample. This kind of metamaterial has already been demonstrated by theoretical and experimental researches based on liquid crystals and  $\Omega$ -shaped microstructures from GHz to visible regions [28–32]. For simplicity, Drude model is chosen as

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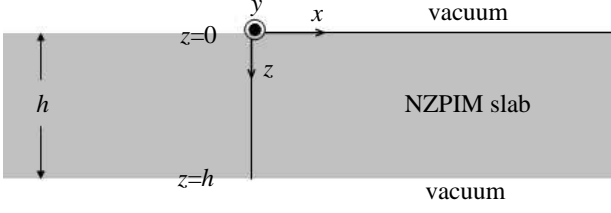


FIG. 1: Schematic of the NZPIM slab in vacuum with thickness  $h$  along  $z$  direction. It is assumed that the slab is infinite along  $x$  and  $y$  direction.

the parameter of the permittivity and the permeability of the NZPIM slab [10, 33]:

$$\epsilon(\omega) = 1 - \frac{\omega_{pe}^2}{\omega^2 + i\gamma_e\omega}, \quad \mu(\omega) = 1 - \frac{\omega_{pm}^2}{\omega^2 + i\gamma_m\omega}, \quad (1)$$

where  $\omega_{pe}$  and  $\omega_{pm}$  are controllable electronic and magnetic plasma frequencies, and  $\gamma_e$  and  $\gamma_m$  are the damping rates. As shown in [30], with suitable structure parameters,  $\omega_p = \omega_{pe} = \omega_{pm}$  can be achieved in a back-to-back  $\Omega$ -shaped meta-material in GHz region. When  $\omega = \omega_p = \omega_{pe} = \omega_{pm}$  and  $\gamma = \gamma_e = \gamma_m \ll \omega_p$ ,  $\epsilon(\omega)$  and  $\mu(\omega)$  are nearly equal to zero, simultaneously. Thus, the effective refractive index of the sample varies from negative, through zero, then to positive with the increase of frequency. In such case, the so-called DP is the zero point of effective refractive index.

We consider a paraxial, coherent, TE-polarized, spatially and temporally Gaussian pulse injecting into the sample along  $z$  direction in vacuum as shown in the Fig. 1. It is assumed that the slab sample is infinite along  $x$  and  $y$  direction. The refractive index is  $n_1(\omega) = \sqrt{\epsilon(\omega)\mu(\omega)} = 1 - \omega_p^2/(\omega^2 + i\gamma\omega)$  and the thickness of the sample is  $h$ . The function of the pulse at the initial plane  $z = z_0 < 0$  [while plane  $z = 0$  is the interface between vacuum ( $n_0 = 1$ ) and the NZPIM slab sample] is given as

$$E_i(x, t, z_0) = \exp\left(-\frac{t^2}{\Gamma^2} - \frac{x^2}{W_0^2}\right) \exp(-i\omega_0 t). \quad (2)$$

This pulse can be decomposed into its monochromatic Fourier components and further be expanded in angular spectra of monochromatic plane-wave components

$$E_i(k_x, \omega, z_0) = \frac{\Gamma W_0}{4\pi} \exp\left[-\frac{\Gamma^2(\omega - \omega_0)^2}{4}\right] \exp\left(-\frac{k_x^2 W_0^2}{4}\right), \quad (3)$$

where  $\Gamma$  is the temporal half-width,  $W_0$  is the spatial half-width,  $k_x = (n_0\omega/c)\sin\theta$ ,  $c$  is the light velocity in vacuum, and  $\omega_0$  is the central angular frequency of the pulse.

The transmission coefficient  $t(\theta, \omega) = |t(\theta, \omega)| \exp[i\varphi(\theta, \omega)]$  of the sample can be deduced from the electromagnetic continuity conditions of the two interfaces (plane  $z = 0$  and  $z = h$  in Fig. 1) [34]. In this paper, for the TE-polarized incident wave,  $t(\theta, \omega)$  can be written as

$$t(\theta, \omega) = \frac{2}{2\cos\delta - i\sin\delta \left( \frac{\cos\theta_1}{\cos\theta} + \frac{\cos\theta}{\cos\theta_1} \right)}. \quad (4)$$

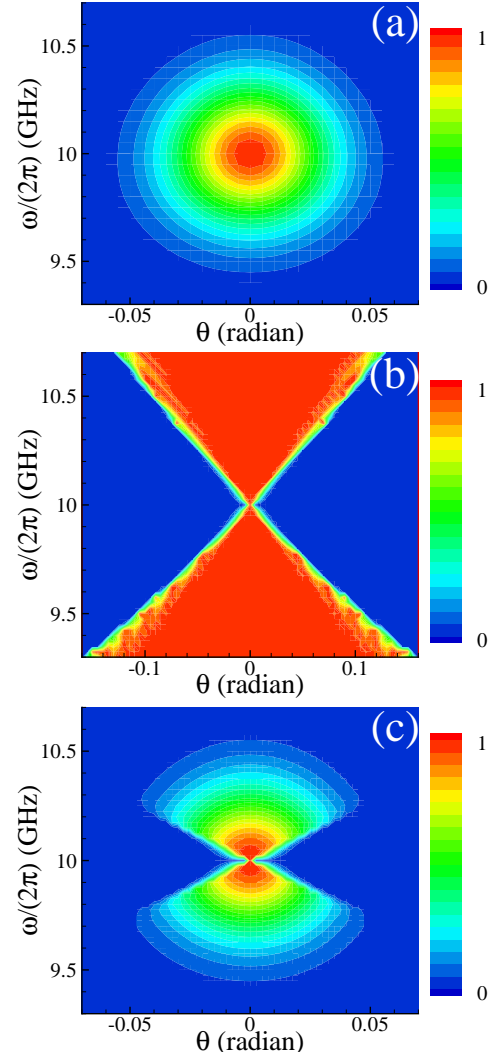


FIG. 2: (Color online) (a) The spatio-temporal spectrum of incident Gaussian pulse with  $\Gamma = 1.0\text{ns}$ ,  $W_0 = 10\lambda_0$  ( $\lambda_0 = 0.03\text{m}$ ), and  $f_0 = 10\text{GHz}$ . (b) Distribution of the spectral transmittance ( $|t(\theta, \omega)|$ ) for a sample with  $h = 40\lambda_0$  and  $\gamma = \gamma_e = \gamma_m = 10^4\text{Hz}$ , the blue part denotes the stop-band and the rest denotes the pass-band. When  $\omega = \omega_p = 2\pi \times 10\text{GHz}$ , the refractive index is zero. (c) The spatio-temporal spectrum of the transmitted pulse.

Here,  $\theta_1 = \arcsin[n_0 \sin\theta / n_1(\omega)]$  is the refractive angle of the angular spectrum component in the sample which can be obtained from Snell's law and  $\delta = \omega n_1(\omega)h \cos\theta_1 / c$  stands for the phase change when electromagnetic wave cross the sample [34].

In this case, the transmission coefficient is equivalent to the frequency response of a linear filter [35–39]. Therefore the spectral function at the exit end can be written as  $E_t(\theta, \omega, z) = E_t(\theta, \omega, 0)t(\theta, \omega)$ .  $E_t(\theta, \omega, 0) = E_i(\theta, \omega, z_0) \exp(-ik_z z_0)$  is the function of spatio-temporal spectrum of the pulse arriving at the plane  $z = 0$ , where  $k_z = k_0 \cos\theta$  ( $k_0 = n_0\omega/c$  is the wave vector in vacuum). We take the spatial and temporal half-width of the incident pulse as  $10\lambda_0$  ( $\lambda_0 = c/f_0$  is central wavelength of the pulse in vacuum) and  $1\text{ns}$ , respectively, with the central frequency  $f_0 (=10\text{GHz})$  for an example. The spatio-

temporal spectrum of the incident pulse and the transmittance spectrum distribution  $|t(\theta, \omega)|$  with slab thickness  $h = 40\lambda_0$  are plotted in Figs. 2(a) and 2(b), respectively. Unambiguously, the output spectrum is tailored by the transmittance spectrum as shown in Fig. 2(c). Some spatio-temporal spectrum components are filtered by the stop-band of the transmittance spectrum. In order to obtain the spatio-temporal shape  $E_t(x, t, z)$  of the transmitted pulse, the inverse Fourier transform of  $E_t(\theta, \omega, z)$  has been calculated. But it is difficult to solve such a complicated expression analytically, so numerical integration method is used here. In our analysis, we only need to consider the temporal shape of the transmitted pulses [we consider  $E_t(x = 0, t, z)$ ] because ZB-like effect is the temporally tailed oscillations.

### III. RESULTS AND DISCUSSIONS

As is shown in Fig. 2(b), the pass-band of the transmittance spectrum gradually shrinks to the zero-refractive-index point from both sides of it, forming a double-cone band. The underlying mechanism is that toward the zero-refractive-index point from both sides, the critical angle of the total reflection at the interface  $z = 0$  decreases, thereby only normal incident angular spectra of the pulse can pass through the sample at the zero-refractive-index point, which can be deduced from Snell's law. Even more importantly, the transmittance spectrum is asymmetrical between negative and positive refractive index regions in respect that  $n_1(\omega) = 1 - \omega_p^2/(\omega^2 + i\gamma\omega)$  is asymmetrical for the two regions. The asymmetrical transmittance spectrum gives an asymmetrical filtering effect to the incident spatio-temporal spectrum. This asymmetry in frequency domain is the origin of the tailed oscillations in time domain. In a sense, a pulse shape with tailed oscillations could be seen as a kind of asymmetry. At the same time, the transmittance spectrum is symmetrical for negative and positive incident angle  $\theta$ , deduced from Eq. (4). Thus, the spatial shape of the pulse is symmetrical (seen Fig. 2 in Ref. [11]). In addition, it is worth noting that the temporally tailed oscillations will also arise when the central frequency of the pulse diverge from the zero-refractive-index point slightly, on condition that the asymmetrical filtering effect still holds.

Equation (4) determines the sample's transmittance spectrum that is a function of the sample thickness  $h$ . As  $h$  increases, the transition region from the pass-band to stop-band becomes steeper and steeper with increasingly severe oscillations, which will influence the filtering effect and, then, the oscillating properties of the transmitted pulse. In Fig. 3(a), we plot the relative intensity  $|E_t(x = 0, t, z)/E_t(x = 0, t, z)_{\max}|^2$  of the transmitted pulses with the sample thickness  $h = 10\lambda_0, 30\lambda_0, 50\lambda_0, 70\lambda_0$ , and  $90\lambda_0$ , respectively, while  $\Gamma = 1.0\text{ns}$  and  $W_0 = 10\lambda_0$  are fixed. The initial center of the pulse is assumed at  $z_0 = -20\lambda_0$ . The oscillating strength (the ratios of relative intensities between the second peak and the first peak) of the tailed oscillations increases with  $h$  while the oscillating period  $T$  (time spacing between the second peak and the first peak) remains nearly unchanged, which can be seen from Figs. 3(b) and 3(c). When  $h$  increases to several hundreds of  $\lambda_0$ , the dispersion effect is not negligible and should seriously distort the

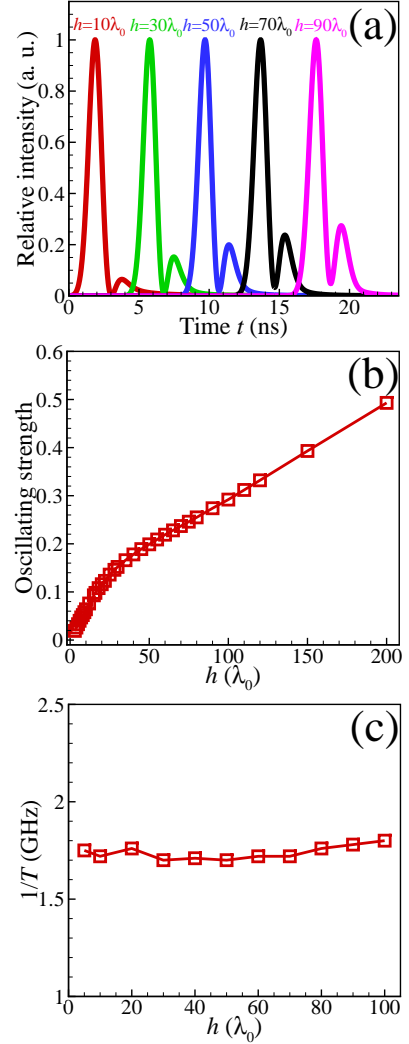


FIG. 3: (Color online) (a) Relative intensity of the transmitted pulse from the calculating data of the inverse Fourier transform with sample thickness  $10\lambda_0, 30\lambda_0, 50\lambda_0, 70\lambda_0$ , and  $90\lambda_0$ , respectively, with the fixed  $\Gamma = 1.0\text{ns}$  and  $W_0 = 10\lambda_0$ . (b) Oscillating strength versus  $h$ . (c) Oscillating frequency  $1/T$  versus  $h$ .

temporal shape of the pulse.

Pulse with different temporal half-width  $\Gamma$  will undergo different responses when propagating in the NZPIM slab sample. The incident spatio-temporal spectrum varies with  $\Gamma$ , thus results in different filtering effects. Near the zero-refractive-index point, the dispersion is considered to be approximately linear [10], however, the linear dispersion approximation is not valid when  $\Gamma$  becomes very small (few optical cycles in time domain but extremely large bandwidth in frequency domain). In Fig. 4(a), we plot the relative intensity of the transmitted pulses with  $\Gamma = 0.5\text{ns}, 1.0\text{ns}$ , and  $1.5\text{ns}$ , respectively, while  $W_0 = 10\lambda_0$  and  $h = 40\lambda_0$  are fixed. Figure 4(b) illustrates the relationship between the oscillating strength and  $\Gamma$ . As  $\Gamma < 0.8\text{ns}$  (only several optical cycles), the oscillating strength varies sharply due to the influence of high-order dispersions. While for  $\Gamma > 0.8\text{ns}$ , the high-order dispersions are negligible, so the oscillating strength has a relatively flat de-

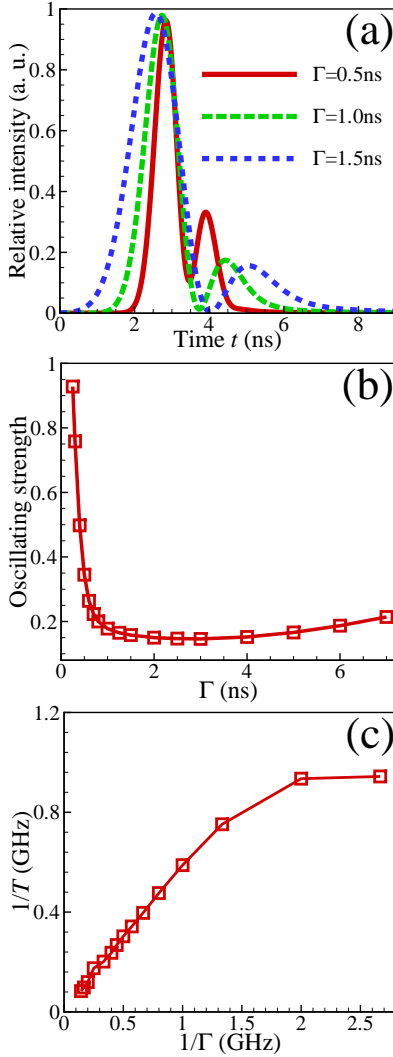


FIG. 4: (Color online) (a) Relative intensity of the transmitted pulse with different temporal half-width 0.5ns, 1.0ns, and 1.5ns, respectively, with  $W_0 = 10\lambda_0$  and  $h = 40\lambda_0$ . (b) Oscillating strength versus  $\Gamma$ . (c) Oscillating frequency  $1/T$  versus  $1/\Gamma$ .

pendence on  $\Gamma$  in the framework of filtering effect alone. The oscillating frequency  $1/T$  increases linearly with the  $1/\Gamma$  under the linear dispersion approximation as shown in Fig. 4(c) (when  $1/\Gamma > 1.25\text{GHz}$ , the dispersion is no longer linear).

Though the ZB-like effect occurs in time domain, the spatial width of the pulse does, indeed, influence the oscillating properties. When  $W_0$  increases, the spatial part of the incident spatio-temporal spectrum becomes narrower and narrower ( $\theta$ -axis direction in Fig. 2(a)). If we consider the limiting situation, that is,  $W_0$  tends to infinity, the pulse only has the angular spectra in normal incident direction, under the circumstances, no spectrum component should be filtered by the transmittance spectrum. So the temporally tailed oscillations would be too weak (nearly vanished) to observe as long as  $W_0$  becomes enough large. In Fig. 5(a), we plot the relative intensity of the transmitted pulses with  $W_0 = 10\lambda_0$ ,  $30\lambda_0$ , and  $50\lambda_0$ , respectively, while  $\Gamma = 1\text{ns}$  and  $h = 40\lambda_0$  are fixed. Seen from Fig. 5(b), we know the oscillating strength decreases with the

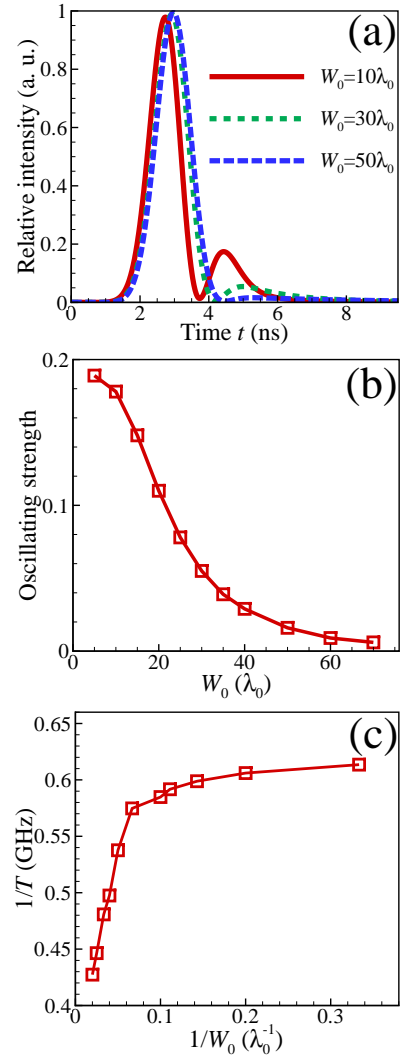


FIG. 5: (Color online) (a) Relative intensity of the transmitted pulse with different spatial half-width  $10\lambda_0$ ,  $30\lambda_0$ , and  $50\lambda_0$ , respectively, with  $\Gamma = 1\text{ns}$  and  $h = 40\lambda_0$ . (b) Oscillating strength versus  $W_0$ . (c) Oscillating frequency  $1/T$  versus  $1/W_0$ .

$W_0$ . The oscillating frequency  $1/T$  raises sharply with small values of  $1/W_0$  and more flatly after coming up to a certain value (about 0.06, see Fig. 5(c)).

It should be pointed out that, although we have only discussed the optical ZB-like effect in metamaterial in the above analysis, the similar phenomena in two-dimensional photonic crystals could be interpreted in the same way. In a recent publication [4], an extremely similar transmittance spectrum in two-dimensional hexagonal photonic crystals near the DP has been revealed.

#### IV. CONCLUSIONS

In conclusion, based on the spatio-temporal filtering analysis, we have presented a physical explanation for the ZB-like effect in NZPIM. When a symmetrical pulse passes through a NZPIM slab near the zero-refractive-index point, its spatio-

temporal spectrum will be filtered asymmetrically for negative and positive refractive index regions, which results in the ZB-like effect in time domain. The strength of tailed oscillations depends on the sample thickness  $h$ , temporal half-width  $\Gamma$ , and the spatial half-width  $W_0$  while the oscillating frequency only relates to  $\Gamma$  and  $W_0$ , and do not change with  $h$ . The results we have got are very similar to what the authors of Refs. [8] and [11] obtained based on the concepts of relativistic quantum mechanics, however, our analysis is a classical method which is irrelevant to the relativistic quantum mechanics.

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